VI. *An Account of a Book intituled,* A Trea-
tife of Fluxions, in Two Books, *by* Colin
Mc Laurin, *A. M. Profeffor of Mathe-
matics in the Univerfity of* Edinburgh,
and Fellow of the ROYAL SOCIETY, 4to.
in Two Volumes, Pages 763.

Prefented Jan.
27. 1742-3.

THE Author's firſt Defign, in com·
poſing this Treatife, was to eſta-
bliſh the Method of Fluxions on Principles equally
evident and unexceptionable with thoſe of the antient
Geometricians, by Demonſtrations deduced after their
Manner, in the moſt rigid Form, and by illuſtrating
the more abſtrufe Parts of the Doctrine, to vindicate
it from the Imputation of Uncertainty or Obſcurity.
But he has likewife comprehended in this Work the
Application of Fluxions to the moſt important geo-
metrical and philofophical Inquiries. It conſiſts of
an Introduction, and Two Books. In the Introduction
he gives an Abſtract of the Difcoveries of the An-
tients in the higher Parts of Geometry, with Obſer-
vations on their Method, and thoſe that firſt ſuc-
ceeded to it. The Firſt Book treats of Fluxions in a
geometrical Method, and the Second treats of the
Computations.

In the Introduction we have an Abſtract not only
of the Difcoveries of the Antients in the higher Parts
of Geometry, but likewife of their Demonſtrations.
After an Account of the Propoſitions of this kind,
that are to be found in the Twelfth Book of *Euclid*,

T t 2   there

there follows a Summary of what is moſt material in the Treatiſes of *Archimedes*, concerning the Sphere and Cylinder, Conoids and Spheroids, the Quadrature of the *Parabola* and the ſpiral Lines. The Demonſtrations are not preciſely in the ſame Form as thoſe of *Archimedes*, but are often illuſtrated from the elementary Propoſitions concerning the Cone, or Corollaries from them, after the Example of *Pappus*, *(Coll. Math.* Prop. 21ſt, Lib. 4.) from whom a Propoſition is demonſtrated, and rendered more general, concerning the Area of the Spiral that is generated on a ſpherical Surface by the Compoſition of Two uniform Motions analogous to thoſe by which the Spiral of *Archimedes* is deſcribed on a Plane. This Area, though a Portion of a curve Surface, is found to admit of a perfect Quadrature, and this Propoſition concludes the Abſtract. He takes occaſion from theſe Theorems to demonſtrate ſome Properties of the Conic Sections, that are not mentioned by the Writers on that Subject; and there are more of this kind deſcribed in the XIth and XIVth Chapters of the Firſt Book.

It is known, that if a Parallelogram, circumſcribed about a given Ellipſe, have its Sides parallel to the conjugate Diameters, then ſhall its Area be of an invariable or given Magnitude, and equal to the Rectangle contained by the Axes of the Figure; but this is only a Caſe of a more general Propoſition. For if, upon any Diameter produced without the Ellipſe, you take Two Points, one on each Side of the Centre at equal Diſtances from it, and the Four Tangents be drawn from theſe Points to the Ellipſe, thoſe Tangents ſhall form a Parallelogram, which is

<div align="right">always</div>

always of a given or invariable Magnitude, when the Ellipfe is given, if the *Ratio* of thofe Diftances to the Diameter be given; and when the *Ratio* of thofe Diftances to the Semidiameter is that of the Diagonal of a Square to the Side, (or of $\sqrt{2 \text{ to } 1}$) the Parallelogram has its Sides parallel to conjugate Diameters. It is likewife fhown here, how the Triangles, *Trapezia*, or Polygons of any kind are determined, which, circumfcribed about a given Ellipfe, are always of a given Magnitude.

There is alfo a general Theorem concerning the *Fruftum* of a Sphere, Cone, Spheroid, or Conoid, terminated by parallel Planes, when compared with a Cylinder of the fame Altitude on a Bafe equal to the middle Section of the *Fruftum* made by a parallel Plane. The Difference betwixt the *Fruftum* and the Cylinder is always the fame in different Parts of the fame, or of fimilar Solids, when the Inclination of the Planes to the Axis, and the Altitude of the *Fruftum*, are given. This Difference vanifhes in the parabolic Conoid. It is the fame in all Spheres; being equal to half the Content of a Sphere of a Diameter equal to the Altitude of the *Fruftum*. In the Cone it is One-fourth of the Content of a fimilar Cone of the fame Height with the *Fruftum*; and in other Figures it is reduced to the Difference in the Cone.

In the Remarks on the Method of the Antients, the Author obferves, that they eftablifhed the higher Parts of their Geometry on the fame Principles as the Elements of the Science, by Demonftrations of the fame kind; that they feem to have been careful not to fuppofe any thing to be done, till by a previous Problem
blem

blem they had shown how it was to be performed: Far less did they suppose any thing to be done, that cannot be conceived to be possible, as a Line or Series to be actually continued to Infinity, or a Magnitude to be diminished till it become infinitely less than it was. The Elements into which they resolved Magnitudes were always finite, and such as might be conceived to be real. Unbounded Liberties have been introduced of late, by which Geometry (wherein every thing ought to be clear) is filled with Mysteries, and Philosophy is likewise perplexed. Several Instances of this kind are mentioned. The Series 1, 2, 3, 4, 5, 6, 7, &c. is supposed by some to be actually continued to Infinity; and, after such a Supposition, we are puzzled with the Question, Whether the Number of finite Terms in such a Series is finite or infinite. In order to avoid such Suppositions, and their Consequences, the Author chose to follow the Antients in their Method of Demonstration as much as possible. Geometry has been always considered as our surest Bulwark against the Subtleties of the Sceptics, who are ready to make use of any Advantages that may be given them against it *; and it is important, not only that the Conclusions in Geometry be true, but likewise that their Evidence be unexceptionable. However, he is far from affirming, that the Method of Infinitesimals is without Foundation, and afterwards endeavours to justify a proper Application of it.

The Grounds of the Method of Fluxions are described in Chap. 1. Book I. and again in Chap. 1.

* See *Bayle*'s Dictionary, Article *Zeno.*

Book

Book II. In the former, Magnitudes are conceived to be generated by Motion, and the Velocity of the generating Motion is the Fluxion of the Magnitude. Lines are suppofed to be generated by the Motion of Points. The Velocity of the Point that defcribes the Line is its Fluxion, and meafures the Rate of its Increafe or Decreafe. Other Magnitudes may be reprefented by Lines that increafe or decreafe in the fame Proportion with them; and their Fluxions will be in the fame Proportion as the Fluxions of thofe Lines, or the Velocities of the Points that defcribe them. When the Motion of a Point is uniform, its Velocity is conftant, and is meafured by the Space which is defcribed by it in a given Time. When the Motion varies, the Velocity at any Term of the Time is meafured by the Space which would be defcribed in a given Time, if the Motion was to be continued uniformly from that Term without any Variation. In order to determine that Space, and confequently the Velocity which is meafured by it, Four Axioms are propofed concerning variable Motions, Two concerning Motions that are accelerated, and Two concerning fuch as are retarded. The Firft is, That the Space defcribed by an accelerated Motion is greater than the Space which would have been defcribed in the fame Time, if it had not been accelerated, but had continued uniform from the Beginning of the Time. The Second is, That the Space which is defcribed by an accelerated Motion, is lefs than the Space which is defcribed in an equal Time by the Motion which is acquired by that Acceleration continued afterwards uniformly. By thefe, and Two fimilar Axioms concerning retarded Motions,

tions, the Theory of Motion is rendered applicable to this Doctrine with the greatest Evidence, without suppofing Quantities infinitely little, or having recourfe to prime or ultimate *Ratios*. The Author firft demonftrates from them all the general Theorems concerning Motion, that are of Ufe in this Doctrine; as that when the Spaces defcribed by Two variable Motions are always equal, or in a given *Ratio*, the Velocities are always equal, or in the fame given *Ratio*; and converfely, when the Velocities of Two Motions are always equal to each other, or in a given *Ratio*, the Spaces defcribed by thofe Motions in the fame Time are always equal, or in that given *Ratio*; that when a Space is always equal to the Sum or Difference of the Spaces defcribed by Two other Motions, the Velocity of the Firft Motion is always equal to the Sum or Difference of the Velocities of the other Motions; and converfely, that when a Velocity is always equal to the Sum or Difference of Two other Velocities, the Space defcribed by the Firft Motion is always equal to the Sum or Difference of the Spaces defcribed by thefe Two other Motions. In comparing Motions in this Doctrine, it is convenient and ufual to fuppofe one of them uniform; and it is here demonftrated, that if the Relation of the Quantities be always determined by the fame Rule or Equation, the *Ratio* of the Motions is determined in the fame manner, when both are fuppofed variable. Thefe Propofitions are demonftrated ftrictly by the fame Method which is carried on in the enfuing Chapters for determining the Fluxions of the Figures.

In

In Chap. II. a Triangle that has Two of its Sides given in Pofition, is fuppofed to be generated by an Ordinate moving parallel to itfelf along the Bafe. When the Bafe increafes uniformly, the Triangle increafes with an accelerated Motion, becaufe its fucceffive Increments are *Trapezia,* that continually increafe. Therefore, if the Motion with which the Triangle flows, was continued uniformly from any Term for a given Time, a lefs Space would be defcribed by it than the Increment of the Triangle which is actually generated in that Time by Axiom I. but a greater Space than the Increment which was actually generated in an equal Time preceding that Term, by Axiom II. and hence it is demonftrated, that the Fluxion of the Triangle is accurately meafured by the Rectangle contained by the correfponding Ordinate of the Triangle, and the right Line which meafures the Fluxion of the Bafe. The Increment which the Triangle acquires in any Time, is refolved into Two Parts; that which is generated in confequence of the Motion with which the Triangle flows at the Beginning of the Time, and that which is generated in confequence of the Acceleration of this Motion for the fame Time. The latter is juftly neglected in meafuring that Motion (or the Fluxion of the Triangle at that Term), but may ferve for meafuring its Acceleration, or the Second Fluxion of the Triangle. The Motion with which the Triangle flows, is fimilar to that of a Body defcending in free Spaces by an uniform Gravity, the Velocity of which, at any Term of the Time, is not to be meafured by the Space defcribed by the Body in a given Time, either before or after that Term, becaufe the

U u   Mo-

Motion continually increafes, but by a Mean between thefe Spaces.

When the Sides of a Rectangle increafe or decreafe with uniform Motions, it may be always confidered as the Sum or Difference of a Triangle and *Trape-zium*; and its Fluxion is derived from the laft Propofition. If the Sides increafe with uniform Motions, the Rectangle increafes with an accelerated Motion; and in meafuring this Motion at any Term of the Time, a Part of the Increment of the Rectangle, that is here determined, is rejected, as generated in confequence of the Acceleration of that Motion.

The Fluxions of a curvilineal Area (whether it be generated by an Ordinate moving parallel to itfelf, or by a Ray revolving about a given Centre) and of the Solid, generated by the Area revolving about the Bafe, are determined by Demonftrations of the fame kind; and when the Ordinates of the Figure increafe, the Increment of the Area is refolved in like manner into Two Parts, one of which is only to be retained in meafuring the Fluxion of the Area, the other being rejected as generated in confequence of the Acceleration of the Motion with which the Figure flows. An Illuftration of Second and Third Fluxions is given by refolving the Increment of a Pyramid or Cone into the feveral refpective Parts that are conceived to be generated in confequence of the Firft, Second, and Third Fluxions of the Solid, when the Axis is fuppofed to flow uniformly.

In Chap. V. a Series of Lines in Geometrical Progreffion are reprefented by an eafy Conftruction. The Firft Term being fuppofed invariable, and the Second to increafe uniformly, all the fubfequent

Terms

Terms increafe with accelerated Motions. The Velocities of the Points that defcribe thofe Lines being compared, it is demonftrated from the Axioms by common Geometry, that the Fluxions of any Two Terms are in a *Ratio* compounded of the *Ratio* of the Terms, and of the *Ratio* of the Numbers that exprefs how many Terms precede them in the Progreffion.

In the VIth Chapter, the Nature and Properties of Logarithms are defcribed after the celebrated Inventor; and it is obferved, that he made ufe of the very Terms *Fluxus* and *Fluat* on this Occafion. A Line is faid to increafe or decreafe *proportionally*, when the Velocity of the Point, that defcribes it, is always as its Diftance from a certain Term of the Line; and if in the mean time another Point defcribes a Line with a certain uniform Motion, the Space defcribed by the latter Point is always the Logarithm of the Diftance of the former from the given Term. Hence the Fluxion of this Diftance is to the Fluxion of its Logarithm as that Diftance is to an invariable Line; and the Fluxions of the Quantities that have their Logarithms in an invariable *Ratio,* are to each other in a *Ratio* compounded of this invariable *Ratio,* and of the *Ratio* of the Quantities themfelves. Some Propofitions are demonftrated, that relate to the Computation of Logarithms; but this Subjeft is profecuted farther in the Second Book. The Logarithmic Curve is here defcribed, with the Analogy betwixt Logarithms and Hyperbolic *Ratios.*

In the VIIth Chapter, after a general Definition of Tangents, it is demonftrated, that the Fluxions of the Bafe, Ordinate, and Curve, are in the fame Propor-

tion

tion to each other, as the Sides of a Triangle respectively parallel to the Base, Ordinate, and Tangent. When the Base is suppofed to flow uniformly, if the Curve be convex towards the Base, the Ordinate and Curve increafe with accelerated Motions; but their Fluxions at any Term are the fame as if the Point which defcribes the Curve had proceeded uniformly from that Term in the Tangent there. Any further Increment which the Ordinate or Curve acquires, is to be imputed to the Acceleration of the Motions with which they flow. A Ray that revolves about a given Centre, being fuppofed to meet any Curve and an Arc of a Circle defcribed from the fame Centre, the Fluxions of the Ray, Curve, and circular Arc, are compared together; and feveral other Propofitions concerning Tangents are demonftrated from the Axioms. The next Chapter treats of the Fluxions of curve Surfaces in a fimilar manner.

The IXth Chapter treats chiefly of the greateft and leaft Ordinates of Figures, and of the Points of contrary Flexure and Cufpids. The Fluxion of the Bafe being given, when the Fluxion of the Ordinate vanifhes, the Tangent becomes parallel to the Bafe, and the Ordinate moft commonly is a *Maximum* or *Minimum*, according to the Rule given by Authors upon this Subject. But if the Second Fluxion of the Ordinate vanifh at the fame time, and the Third Fluxion be real, this Rule does not hold, for the Ordinate is in that Cafe neither a *Maximum* nor *Minimum*. If the Firft, Second, and Third Fluxions vanifh, and the Fourth Fluxion be real, the Ordinate is a *Maximum* or *Minimum*. The general Rule demonftrated in this Chapter, and again in the laft Chapter of the

Second

Second Book, is, that when the First Fluxion of the Ordinate, with its Fluxions of any subsequent successive Orders, vanish, and the Number of all these Fluxions that vanish is odd, then the Ordinate is a *Maximum* or *Minimum*, according as the Fluxion of the next Order to these is negative or positive. The Ordinate passes through a Point of contrary Flexure, when its Fluxion becomes a *Maximum* or *Minimum*, supposing the Curve to be continued on both Sides of the Ordinate. Hence the common Rule for finding the Points of contrary Flexure is corrected in a similar manner. Such a Point is not always formed when the Second Fluxion of the Ordinate vanishes; for if its Third Fluxion likewise vanishes, and its Fourth Fluxion be real, the Curve may have its Cavity turned all one Way. The same is to be said, when its Fluxions of the subsequent successive Orders vanish, if the Number of all those that vanish be even. Other Theorems are subjoined relating to this Subject.

The Xth Chapter treats of the Asymptotes of Lines, the Areas bounded by them and the Curves, the Solids generated by these Areas, of spiral Lines, and the Limits of the Sums of Progressions. The Analogy there is betwixt these Subjects, induced the Author to treat of them in one Chapter, and illustrate them by one another. He begins with Three of the most simple Instances of Figures that have Asymptotes. In the common Hyperbola, the Ordinate is reciprocally as the Base, and therefore decreases while the Base increases, but never vanishes, because the Rectangle contained by it and the Base is always a given Area, and it is assignable at any assignable Distance, how

how great foever. The Points of the Conchoid are determined by drawing right Lines from a given Centre, and upon thefe produced from the Afymptote, taking always a given right Line; fo that the Curve never meets the Afymptote, but continually approaches to it, becaufe of the greater and greater Obliquity of this right Line. The Third is the Logarithmic Curve, wherein the Ordinates, at equal Diftances, decreafe in Geometrical Proportion, but never vanifh, becaufe each Ordinate is in a given *Ratio* to the preceding Ordinate. Geometrical Magnitude is always underftood to confift of Parts; and to have no Parts, or to have no Magnitude, are confidered as equivalent in this Science *. There is, however, no Neceffity for confidering Magnitude as made up of an infinite Number of fmall Parts; it is fufficient, that no Quantity can be fuppofed to be fo fmall, but it may be conceived to be diminifhed further; and it is obvious, that we are not to eftimate the Number of Parts that may be conceived in a given Magnitude, by thofe which in particular determinate Circumftances may be actually perceived in it by Senfe; fince a greater Number of Parts become vifible in it by varying the Circumftances in which it is perceived.

It is hardly poffible to give a tolerable Extract of this or the following Chapters, without Diagrams and Computations: We fhall therefore obferve only, that after giving fome plain and obvious Inftances, wherein a Quantity is always increafing, and yet never

* See *Euclid's* Elements, Def. I. Lib. I.

amounts to a certain finite Magnitude (as, while the Tangent increafes, the Arc increafes, but never amounts to a Quadrant); this is applied fucceffively to the feveral Subjects mentioned in the Title of the Chapter. Let the Figure be concave towards the Bafe, and fuppofe it to have an Afymptote parallel to the Bafe; in this Cafe the Ordinate always increafes while the Bafe is produced, but never amounts to the Diftance between the Afymptote and the Bafe. In like manner a curvilineal Area, in a Second Figure, may increafe, while the Bafe is produced, and approach continually to a certain finite Space, but never amount to it: This is always the Cafe, when the Ordinate of this latter Figure is to a given right Line, as the Fluxion of the Ordinate of the former is to the Fluxion of the Bafe; and of this various Examples are given. A Solid may increafe in the fame manner, and yet never amount to a given Cube or Cylinder, when the Square of the Ordinate of the latter Figure is to a given Square, as the Fluxion of the Ordinate of the firft Figure is to the Fluxion of the Bafe. A Spiral may in like manner approach to a Point continually, and yet in any Number of Revolutions never arrive at it; and there are Progreffions of Fractions that may be continued at Pleafure, and yet the Sum of the Terms may be always lefs than a given Number. Various Rules are demonftrated, and illuftrated by Examples, for determining when a Figure has an Afymptote parallel or oblique to the Bafe; when the Area terminated by the Curve and the Afymptote has a Limit which it never exceeds, or may be produced till it furpafs any affignable Space; when the Solid generated by that Area, the

the Surface generated by the Perimeter of the Curve, the fpiral Area generated by the revolving Ray, the fpiral Line itfelf, or the Sum of the Terms of a Progreffion, have fuch Limits or not; and for meafuring thofe Limits. The Author infifts on thefe Subjects, the rather that they are commonly defcribed in very myfterious Terms, and have been the moft fertile of Paradoxes of any Parts of the higher Geometry. Thefe Paradoxes, however, amount to no more than this: That a Line or Number may be continually acquiring Increments, and thofe Increments may decreafe in fuch a manner, that the whole Line or Number fhall never amount to a given Line or Number. The Neceffity of admitting this is obvious enough, and is here fhewn from the Nature of the moft common geometrical Figures in Art. 292, 293, &c. and from any Series of Fractions that decreafe continually, in Art. 354, 355, &c.

The XIth Chapter treats of the Curvature of Lines, its Variation, the Degrees of Contact of the Curve and Circle of Curvature, and of various Problems that depend on the Curvature of Lines. This Subject is treated fully, becaufe of its extenfive Ufefulnefs, and becaufe in this confifts one of the greateft Advantages of the modern Geometry above that of the Antients. The Author on this, as former Occafions, begins by premifing the neceffary Definitions. Curve Lines touch each other in a Point, when the fame right Line is their common Tangent at that Point; and that which has the clofeft Contact with the Tangent, or paffes betwixt it and the other Curve through the Angle of Contact formed by them, being lefs inflected from the Tangent, is therefore lefs curve. Thus a

greater

greater Circle has a lefs Curvature than a leſſer Circle; and ſince the Curvature of Circles may be varied indefinitely, by inlarging or diminiſhing their Diameters, they afford a Scale by which the Curvature of other Lines may be meaſured. As the Tangent is the right Line which touches the Arc ſo cloſely, that no other right Line can be drawn between them; ſo the Circle of Curvature is that which touches the Curve ſo cloſely, that no other Circle can be drawn through the Point of Contaƈt between them. As the Curve is ſeparated from its Tangent in conſequence of its Flexure or Curvature, ſo it is ſeparated from the Circle of Curvature in conſequence of the Variation of its Curvature; which is greater or lefs, according as its Flexure from that Circle is greater or lefs.

The Tangent of the Figure being conſidered as the Baſe, a new Figure is imagined, whoſe Ordinate is a Third Proportional to the Ordinate and Baſe of the Firſt. This new Figure determines the Chord of the Circle of Curvature by its Interſeƈtion with the Ordinate at the Point of Contaƈt, and by the Tangent of the Angle in which it cuts that Circle, meaſures the Variation of Curvature. The lefs this Angle is, the cloſer is the Contaƈt of the Curve and Circle of Curvature, of which there may be indefinite Degrees. When the Figure propoſed is a conic Seƈtion, the new Figure is likewiſe a conic Seƈtion; and it is a right Line when the Firſt Figure is a *Parabola*, and the Ordinates are parallel to the Axis; or when the Firſt Figure is an *Hyperbola*, and the Ordinates are parallel to either Aſymptote. Hence the Curvature and its Variation in a conic Seƈtion are determined by ſeveral Conſtruƈtions; and, amongſt other Theorems,

X x

orems, it is ſhewn, that the Variation of Curvature at any Point of a conic Section is as the Tangent of the Angle contained by the Diameter which paſſes through that Point, and by the Perpendicular to the Curve.

When the Ordinate at the Point of Contact is an Aſymptote to the new Figure, the Curvature is leſs than in any Circle; and this is the Caſe in which it is ſaid to be infinitely little, or the Ray of Curvature is ſaid to be infinitely great. Of this kind is the Curvature at the Points of contrary Flexure in the Lines of the Third Order. When the new Figure paſſes through the Point of Contact, the Curvature is greater than in any Circle, or the Ray of Curvature vaniſhes; and in this Caſe the Curvature is ſaid to be infinitely great. Of this kind is the Curvature at the Cuſpids of the Lines of the Third Order.

As Lines which paſs through the ſame Point have the ſame Tangent when the Firſt Fluxions of the Ordinate are equal, ſo they have the ſame Curvature when the Second Fluxions of the Ordinate are likewiſe equal; and half the Chord of the Circle of Curvature that is intercepted between the Points wherein it interſects the Ordinate, is a Third Proportional to the right Lines that meaſure the Second Fluxion of the Ordinate and Firſt Fluxion of the Curve, the Baſe being ſuppoſed to flow uniformly. When a Ray revolving about a given Point, and terminated by the Curve, becomes perpendicular to it, the Firſt Fluxion of the Ray vaniſhes; and if its Second Fluxion vaniſhes at the ſame time, that Point muſt be the Centre of Curvature. The ſame is to be ſaid when the angular Motion of the Ray about that Point

Point is equal to the angular Motion of the Tangent of the Curve; as the angular Motion of the *Radius* of a Circle about its Centre is always equal to the angular Motion of the Tangent of the Circle. Thus the various Properties of the Circle fuggeſt various Theorems for determining the Centre of the Curvature.

Becauſe Figures are often ſuppoſed to be deſcribed by the Interſections of right Lines revolving about given Poles, Three Theorems are given in Prop. 18. 26. and 35. for determining the Tangents, Aſymptotes, and Curvature of ſuch Lines, from the Deſcription, which are illuſtrated by Examples. A new Property of Lines of the Third Order is ſubjoined to Prop. 35. The Evolution of Lines is conſidered in Prop. 36. The Tangents of the *Evoluta* are the Rays of Curvature of the Line which is deſcribed by its Evolution; and the Variation of Curvature in the latter is meaſured by the *Ratio* of the Ray of Curvature of the former to the Ray of Curvature of the latter.

Sir *Iſaac Newton*, in a Treatiſe lately publiſhed, meaſures the Variation of the Curvature by the *Ratio* of the Fluxion of the Ray of Curvature to the Fluxion of the Curve; and is followed by the Author, to avoid the Perplexity which a Difference in Definitions occaſions to Readers, though he hints (in Art. 386.) that this *Ratio* gives rather the Variation of the Ray of Curvature, and that it might have been proper to have meaſured the Variation of Curvature rather by the *Ratio* of the Fluxion of the Curvature itſelf to the Fluxion of the Curve; ſo that the Curvature being inverſely as the Ray of Curvature, and

X x 2   con-

confequently its Fluxion as the Fluxion of the Ray itfelf directly, and the Square of the Ray inverfely, its Variation would have been directly as the Meafure of it, according to Sir *Ifaac Newton's* Definition, and inverfely as the Square of the Ray of Curvature: According to this Explication, it would have been meafured by the Angle of Contact contained by the Curve and Circle of Curvature, in the fame manner as the Curvature itfelf is meafured by the Angle of Contact contained by the Curve and Tangent. The Ground of this Remark will better appear from an Example: According to Sir *Ifaac Newton's* Explication, the Variation of Curvature is uniform in the Logarithmic Spiral, the Fluxion of the Ray of Curvature in this Figure being always in the fame *Ratio* to the Fluxion of the Curve; and yet while the Spiral is produced, though its Curvature decreafes, it never vanifhes; which muft appear ftrange to fuch as do not attend to the Import of his Definition.—— It is eafy, however, to derive one of thefe Meafures of this Variation from the other, and becaufe Sir *Ifaac Newton's* is (generally fpeaking) affigned by more fimple Expreffions, the Author has the rather conformed to it in this Treatife, but thought it neceffary to give the Caution we have mentioned.

The greateft Part of this Chapter is imployed in treating of ufeful Problems, that have a Dependence on the Curvature of Lines. Firft, the Properties of the Cycloid are briefly demonftrated, with the Application of this Doctrine to the Motion of Pendulums, by fhewing that when the Motion of the generating Circle along the Bafe is uniform, and therefore may meafure the Time, the Motion of the Point that de-

defcribes the Cycloid, is fuch as would be acquired by
a heavy Body defcending along the cycloidal Arc,
the Axis of the Figure being fuppofed perpendicular
to the Horizon. In the next place, the Cauftics, by
Reflexion and Refraction, are determined. If Per-
pendiculars be always drawn from the radiating Point
to the Tangents of the Curve, and a new Curve be
fuppofed to be the *Locus* of the Interfections of the
Perpendiculars and Tangents, then the Line, by the
Evolution of which that new Curve can be defcribed,
is fimilar and fimilarly fituated to the Cauftic by Re-
flexion. The Doctrine of centripetal Forces is treated
at length from Art. 416. to 493.

Firft, a Body is fuppofed to defcend freely by its
Gravity in a vertical Line; and becaufe the Gravity
is the Power which accelerates the Motion of the
Body, it muft be meafured by the Fluxion of its
Velocity, or the Second Fluxion of the Space de-
fcribed by it. When the vertical Line is fuppofed
to move parallel to itfelf with an uniform Motion,
the Body will defcend in it in the fame manner as
before; and the Gravity will be ftill meafured by the
Second Fluxion of the Defcent, or the Second Fluxion
of the Ordinate of the Curve that is traced in this
Cafe by the Body on an immoveable Plain, and there-
fore is as the Square of the Velocity (which is mea-
fured by the Fluxion of the Curve) directly, and the
Chord of the Circle of Curvature that is in the Di-
rection of the Gravity inverfely, by a Propofition
mentioned above. When the Gravity acts uniformly,
and in parallel Lines, the Projectile, in defcribing any
Arc, falls below the Tangent drawn at the Beginning
of the Arc, as much as if it had fallen perpendicu-
larly

larly in the Vertical; and the Time being given, the Gravity may be meafured by the Space which is the Subtenfe of the Angle of Contact. In other Cafes, when the Gravity varies, or its Direction changes, it may be meafured at any Point by the Subtenfe of the Angle of Contact, that would have been generated in a given Time, if the Gravity had continued to act uniformly in parallel Lines from that Term, that is, by the Subtenfe of the Angle of Contact in the Parabola that has its Diameter in the Direction of the Force, and has the clofeft Contact with the Curve; which leads us to the fame Theorem as before.

In general, let the Gravity (that refults from the Compofition of any Number of centripetal Forces, which are fuppofed to act on the Body in one Plane) be refolved into a Force parallel to the Bafe; then the former fhall be meafured by the Second Fluxion of the Ordinate, and the latter by the Second Fluxion of the Bafe, the Time being fuppofed to flow uniformly, fo that the Velocity of the Body may be meafured by the Fluxion of the Curve. When the Trajectory is not in one Plane, the Force is refolved in a fimilar manner into Three Forces, which are meafured by Three Second Fluxions analogous to them.

Whether the Body move in a Void, or in a Medium that refifts its Motion; the Gravity that refults from the Compofition of the centripetal Forces which act upon the Body, is always as the Square of its Velocity directly, and the Chord of the Circle of Curvature that is in the Direction of the Gravity inverfely.

When

When a Body defcribes any Trajectory in a Void or in a Medium, by a Force directed to One given Centre, the Velocity at any Point of the Trajectory is to the Velocity by which a Circle could be defcribed in a Void about the fame Centre, at the fame Diftance, by the fame Gravity, in the fubduplicate *Ratio* of the angular Motion of the Ray drawn always from the Body to the Centre, to the angular Motion of the Tangent of the Trajectory : And, if there be no Refiftance, the Velocity in the Trajectory at any Point, is the fame that would be acquired by the Body, if it was to fall from that Point through One-fourth of the Chord of the Circle of Curvature that is in the Direction of the Gravity, and the Gravity at that Point was to be continued uniformly during its Defcent.

If the centripetal Force be inverfely as any Power of the Diftance whofe Exponent is any Number m greater than Unit, there is a certain Velocity *(viz.* that which is to the Velocity in a Circle at the fame Diftance as $\sqrt{2}$ to $\sqrt{m-1}$) which would be juft fufficient to carry off the Body upwards in a vertical Line, fo as that it fhould continue to afcend for ever, and never return towards the Centre. If the Body be projected in any other Direction with the fame Velocity, it will defcribe a Trajectory which is here conftructed : It is a *Parabola* when $m=2$, a Logarithmic Spiral when $m=3$, an Epicyloid when $m=4$, a Circle that paffes through the Centre of the Forces when $m=5$, and the *Lemnifcata* when $m=7$. In general, it is conftructed by drawing a Perpendicular from the Centre of the Forces to a right Line given in Pofition, and any other Ray to the fame right Line,

then

then increafing or diminifhing the Angle contained by this Ray and the Perpendicular in the given *Ratio* of 2 to the Difference between 3 and *m*, and increafing or diminifhing the Logarithm of the Ray in the fame given *Ratio.* The Trajectories defcribed in analogous Cafes by centrifugal Forces, are conftructed in a fimilar manner. Thefe are the Figures in which the Perpendicular, from a given Centre on the Tangent, is always as fome Power of the Ray drawn from the fame Centre to the Point of Contact, which are afterwards found to arife in the Refolution of the moft fimple Cafes of Problems of various kinds.

When the Area defcribed about the Centre of an Ellipfe is given, the Subtenfe of the Angle of Contact, drawn through one Extremity of the Arc parallel to the Semidiameter drawn to the other Extremity, is in a given *Ratio* to this Semidiameter ; and therefore, when an Ellipfe is defcribed by a Force directed towards the Centre, that Force is always as the Diftance from the Centre. When the Force is directed toward the *Focus*, it is inverfely as the Square of the Diftance. And thefe Two Cafes are confidered particularly, becaufe of their Ufefulnefs in the true Theory of Gravity. To illuftrate which, the Laws of centripetal Forces that would caufe a Body to defcend continually toward the Centre, or afcend from it, are diftinguifhed from thofe which caufe the Body to approach towards the Centre, and recede from it by turns. A Body approaches from the higher Apfid toward the Centre, when its Velocity is lefs than what is requifite to carry it in a Circle; and if its Velocity increafe, while it defcends, in a

higher

higher Proportion than the Velocities requifite to carry Bodies in Circles about the fame Centre, the Velocity in the lower Part of the Curve may exceed the Velocity in a Circle at the fame Diftance, and thereby become fufficient to carry off the Body again. But while the Diftance decreafes, if the Velocities in Circles increafe in the fame or in a higher Proportion, than the Velocity in a Trajectory can increafe, the Body muft either continually approach toward the Centre, if it once begin to approach to it, or recede continually from the Centre, if it once begin to afcend from it; and this is the Cafe, when the centripetal Force increafes as the Cube of the Diftance decreafes, or in a higher Proportion. But though, in fuch Cafes, the Body approach continually towards the Centre, we are not to conclude, that it will always approach to it till it fall into it, or come within any given Diftance; for it is demonftrated afterwards in Art. 879 and 880. that it may approach to the Centre for ever, in a Spiral that never defcends to a given Circle defcribed in the fame Plane, and that it may recede from it for ever in a Spiral that never arifes to a given Altitude. An Example of each Cafe is given when the centripetal Force is inverfely as the Fifth Power of the Diftance.

When the Trajectory is defcribed in a *Medium*, let *z* be to a given Magnitude as the centripetal Force is to the Force by which the fame Trajectory could be defcribed in a Void; and if the Area be fuppofed to flow uniformly, the Refiftance will be in the compound *Ratio* of the Fluxion of *z*, and of the Fluxion of the Curve; and the Denfity of the Medium (fuppofing the Refiftance to be in the compound *Ratio*

of

of the Denfity and of the Square of the Velocity) fhall be as the Fluxion of the Logarithm of *z* directly, and the Fluxion of the Curve inverfely. Hence, when any Figure that can be defcribed in a Void by a Force that varies according to any Power of the Diftance from the Centre, is defcribed in a Medium, the Denfity of the Medium muft be inverfely as the Tangent of the Figure bounded by a Perpendicular at the Centre to the Ray drawn from it to the Point of Contact.

After giving fome Properties of the Trajectories, that are defcribed by a Body when it gravitates in right Lines perpendicular to a given Surface, and their Application to optical Ufes, the Author proceeds to confider the Motion of a Body that gravitates towards feveral Centres. In fuch Cafes, that Surface is faid to be horizontal, which is always perpendicular to the Direction of the Gravity that refults from the Compofition of the feveral Forces; and it is fhewn, that the Velocity which is acquired by defcending from one horizontal Surface to another, is always the fame (whether the Body move in right Lines, or in any Curves); the Square of which is meafured by the Aggregate of feveral Areas which have the Diftances from the refpective Centres for their Bafes, and right Lines proportional to the Forces at thefe Diftances for their Ordinates.

The Force which acts upon the Moon is refolved into a Force perpendicular to the Plane of the Ecliptic, and a Force parallel to it. This laft is again refolved into that which is parallel to the Line of the *Syzigies*, and that which is parallel to the Line joining the Quadratures. The Firft meafures the Se-

cond

cond Fluxion of the Diſtance of the Moon from that Plane, the Second and Third meaſure the Second Fluxions of her Diſtances from the Line of the Quadratures, and from the Line of the *Syzigies*, reſpectively. Hence a Conſtruction is derived of the Trajectory which would be deſcribed by the Moon about the Earth, in conſequence of their unequal Gravitation towards the Sun, if the Gravity of the Moon towards the Earth was as her Diſtance from it. From this a Computation is deduced of the Motion of the Nodes of the Moon, and of the Variation of the Inclination of the Plane of her Orbit, which we cannot deſcribe here. It is ſufficient to obſerve, that theſe Motions are found to agree nearly with thoſe which have been deduced from other Theories, and from Aſtronomical Obſervations.

A Fluid being ſuppoſed to gravitate towards two given Centres with equal and invariable Forces, it is ſhewn, that the Figure of the Fluid muſt be that of an oblong Spheroid, and that thoſe two Centres muſt be the *Foci* of the generating Ellipſe. The Nature of the Figure is alſo ſhewn, when the Fluid gravitates towards ſeveral Centres, or when it revolves on its Axis; but theſe are mentioned briefly, becauſe ſuch Theories are of little or no Uſe for diſcovering the Figures of the Planets.

In the 12th Chapter, the Author proceeds to conſider the more conciſe Methods, by which the Fluxions of Quantities are uſually determined, and to deduce general Theorems more immediately applicable to the Reſolution of Geometrical and Philoſophical Problems. In the Method of Infiniteſimals, the Element, by which any Quantity increaſes or decreaſes,

is

is fuppofed to become infinitely fmall, and is gene-
rally expreffed by Two or more Terms, fome of
which become infinitely lefs than the reft, and there-
fore being neglected as of no Importance, the re-
maining Terms form, what is called the *Difference*
of the Quantity propofed. The Terms that are neg-
lected in this manner are the very fame which arife
in confequence of the Acceleration or Retardation
of the generating Motion, during the infinitely fmall
Time in which the Element is generated ; and there-
fore thefe Differences are in the fame *Ratio* to each
other as the generating Motions or Fluxions. Hence
the Conclufions in this Method are accurately true,
without even an infinitely fmall Error, and agree
with thofe that are deduced by the Method of
Fluxions.

It is ufual in this Method to confider a Curve as a
Polygon of an infinite Number of Sides, which, being
produced, give the Tangents of the Curve, and, by
their Inclination to each other, meafure its Cur-
vature. But it is neceffary in fome Cafes, if we
would avoid Error, to refolve the Element of the
Curve into feveral infinitely fmall Parts, or even
fometimes into Infinitefimals of the Second Order ;
and Errors that might otherwife arife in its Applica-
tion, may, with due Care, be corrected by a proper
Ufe of this Method itfelf, of which fome Inftances
are given. If we were to fuppofe, for Example, the
leaft Arc that can be defcribed by a Pendulum to
coincide with its Chord, the Time of the Vibration
derived from this Suppofition will be found er-
roneous; but by refolving that Arc into more and
more infinitely fmall Parts, we approach to the true
Time

Time in which it is defcribed. By fuppofing the Tangent of the Curve to be the Production of the rectilineal Element of the Curve, the Subtenfe of the Angle of Contact is found equal to the Second Difference or Fluxion of the Ordinate; but in this Inquiry, the Tangent ought to be fuppofed to be equally inclined to the two Elements of the Curve that terminate at the Point of Contact; and then the Subtenfe of the Angle of Contact will be found equal to half the Second Difference of the Ordinate, which is its true Value.

Sir *Ifaac Newton*, however, inveftigates the Fluxions of Quantities in a more unexceptionable manner. He firft determines the finite fimultaneous Increments of the Fluents, and, by comparing them, inveftigates the *Ratio* that is the Limit of the various Proportions which they bear to each other, while he fuppofes them to decreafe together till they vanifh. When the generating Motions are variable, the *Ratio* of the fimultaneous Increments that are generated from any Term, is expreffed by feveral Quantities, fome of which arife from the *Ratio* of the generating Motions at that Term, and others from the fubfequent Acceleration or Retardation of thefe Motions. While the Increments are fuppofed to be diminifhed, the former remain invariable, but the latter decreafe continually, and vanifh with the Increments; and hence the Limit of the variable *Ratio* of the Increments (or their ultimate *Ratio)* gives the precife *Ratio* of the generating Motions or Fluxions. Moft of the Propofitions in the preceding Chapters may be more briefly demonftrated by this Method, (of which

feveral

several Examples are given) and the Author makes always use of it in the Sequel of this Book.

It is one of the great Advantages of this Method, that it suggests general Theorems for the Resolution of Problems, which may be readily applied as there is occasion for them. Our Author proceeds to treat of these, and first of such as relate to the Centre of Gravity and its Motion. In any System of Bodies, the Sum of their Motions, estimated in a given Direction, is the same as if all the Bodies were united in their common Centre of Gravity. If the Motion of all the Bodies is uniform and rectilineal, the Centre of Gravity is either quiescent, or its Motion is uniform and rectilineal. When Action is equal to Reaction, the State of the Centre of Gravity is never affected by the Collisions of the Bodies, or by their attracting or repelling each other mutually. It is not, however, the Sum of the absolute Motions of the Bodies that is preserved invariable in consequence of the Equality of the Action and Reaction, as they seem to imagine, who tell us, that this Sum is unalterable by the Collisions of Bodies, and that this follows so evidently from the Equality of Action and Reaction, that to endeavour to demonstrate it would serve only to render it more obscure. On this Occasion the Author illustrates an Argument which he had proposed in a Piece that obtained the Prize proposed by the *Royal Academy of Sciences* at *Paris* in 1724. against the Mensuration of the Forces of Bodies by the Square of the Velocities, shewing that if this Doctrine was admitted, the same Power or Agent, exerting the same Effort, would produce more Force in the same Body when in a Space carried uniformly
forwards,

forwards, than if the Space was at Reft; or that Springs acting equally on Two equal Bodies in such a Space, would produce unequal Changes in the Forces of thofe Bodies.

Various Problems concerning the Collifion of Bodies are refolved in a more general manner than ufual. Mr. *Bernouilli* had determined the Motions when the Elafticity is perfect, and One Body ftrikes Two equal Bodies in Directions that form equal Angles with its Direction; or when there are any Number of Bodies impelled by it on one Side in various Directions, providing equal Bodies be impelled by it on the other Side, in Directions equally inclined to its own Direction. But the Problem is refolved here without thefe Limitations; fome others of this kind are fubjoined, and this Doctrine is applied for determining the Motions of Bodies that act upon each other while they defcend by their Gravity.

The general Principle derived from thefe Inquiries is, that if there be no Collifion, or fudden Communication of Motion from one Body to another, while they defcend together, and in any cafe, if the Elafticity be perfect, the Sum of the Products, when each Body is multiplied by the Square of the Velocity acquired by it, is the fame as if all the Bodies had defcended freely from the fame refpective Altitudes to their feveral Places; only in collecting that Sum, if any Body is made to afcend, the Product of it multiplied by the Square of its Velocity is to be fubducted: And if the Bodies be fuppofed to afcend from their Places with the refpective Velocities acquired by them, then their common Centre of Gravity will rife to the
<div align="right">fame</div>

fame Level from which it defcended. In other Cafes, however, the Afcent of the Centre of Gravity will be lefs than its Defcent, but is never greater.

After demonftrating the ufual Rule for finding the Centre of Ofcillation, the Author treats of the Motion of Water iffuing from a cylindric Veffel. The Effect of the Gravitation of the whole Mafs of Water is confidered as Threefold. It accelerates, for fome time at leaft, the Motion with which the Water in the Veffel defcends; it generates the Excefs of the Motion with which the Water iffues at the Orifice above the Motion which it had in common with the reft of the Water; and it acts on the Bottom of the Veffel at the fame time. Then fuppofing the laft Two Parts of the Force to be in any invariable *Ratio* to each other, when the Diameters of the Bafe and Orifice are given, he determines by Logarithms the Velocity with which the Water iffues at the Orifice; and fhews that this Velocity will approach very near to its utmoft Limit in an exceeding fmall Time. When the Water is fuppofed to be fupplied in a Cylinder, fo as to ftand always at the fame Altitude above the Orifice, there is an Analogy between the Acceleration of the Motion of the Water that iffues at the Orifice, and the Acceleration of a Body that defcends by its Gravity in a Medium which refifts in the duplicate *Ratio* of the Velocity. For when the utmoft Velocities, or Limits, are equal in thofe two Cafes, the Time in which the iffuing Water acquires any leffer Velocity, is to the Time in which the defcending Body acquires the fame Velocity as the Area of the Orifice to the Area of the Bafe; and if a cylindric Column be fuppofed to

be

be erected on the Orifice equal to the Quantity of Water that issues at the Orifice in the former of those Times, the Height of this Column will be to the Space described by the descending Body in the latter Time, in the same *Ratio* as the Orifice to the Area of the Base. The *Ratio* of the Force that acts on the Bottom of the Vessel to the Force that generates the Motion of the Water issuing at the Orifice, is deduced from Sir *Isaac Newton*'s Cataract, and is the same that follows from the Principle concerning the Equality of the Ascent and Descent of the Centre of Gravity, which was first applied to this Inquiry by Mr. *Daniel Bernouilli* Comment. *Acad. Petrop. Tom.* 2. But there are several Precautions to be taken in applying this Doctrine.

After some other Theorems concerning the Centre of Gravity, and several Observations concerning the Curvature of Lines, and the Angles of Contact; the Author reprefents four general Propofitions in one View, that the Analogy between them may appear. The Firft gives the Property of the Trajectories that are defcribed by any centripetal Forces, how variable foever thefe Forces, or their Directions, may be. The Second gives a like general Property of the Lines of fwifteft Defcent. The Third gives the Property of the Line that is defcribed in lefs time than any other of an equal Perimeter. And the Fourth gives the Property of the Figure that is affumed by a flexible Line or Chain, in confequence of any fuch Forces acting upon it. If we fuppofe a Body to fet out from any Point in the Trajectory, or in the Line of fwifteft Defcent, with the Velocity which it has acquired there, and to move in the right Line which is the

Z z
Di-

Direction of the Gravity, that refults from the Compofition of the centripetal Forces, then fhall its Velocity, and its Diftance from the Point where the Perpendicular from the Centre of Curvature meets that right Line, flow *proportionally*, *i. e.* the Fluxion of the Velocity (or of the right Line that meafures it) fhall be to the Velocity as the Fluxion of that Diftance is to the Diftance. When the Velocity and Direction of the Motion is the fame in the Line of fwifteft Defcent as in the Trajectory, their Curvature is the fame. Thus in the common Hypothefis of Gravity, the Curvature in the Cycloid, the Line of fwifteft Defcent, is the fame as the *Parabola* defcribed by a Projectile, if the Velocities in thofe Lines be equal, and their Tangents be equally inclined to the Horizon. In order to find the Nature of the *Catenaria* in any Hypothefis of Gravity, fuppofe the Gravity to be increafed or diminifhed in the fame Proportion as the Thicknefs of the Chain varies, and to have its Direction changed into the oppofite Direction; then imagine a Body to fet out with a juft Velocity from a given Point in the Chain, and to defcribe the Curve. The Tenfion of the Chain at any Point will be always as the Square of the Velocity acquired at that Point, and if a Body be projected with this Velocity in the Direction of the Tangent, the Curvature of the Trajectory defcribed by it will be one Half of the Curvature of the Chain at that Point. We muft refer to the Book for a fuller Account of thefe and of other Theorems.

In the XIIIth Chapter, the Problems concerning the Lines of fwifteft Defcent, the Figures which amongft all thofe that have equal Perimeters produce

Ma-

Maxima or *Minima*, and the Solid of leaft Refiftance, are refolved without Computations, from the firft Fluxions only. There are alfo eafy fynthetic Demonftrations fubjoined, becaufe this Theory is commonly efteemed of an abftrufe Nature, and Miftakes have been more frequently committed in the Profecution of it, than of any other relating to Fluxions. To give fome Idea of the Author's Method, fuppofe the Gravity to act in parallel Lines, *a* to denote the Velocity acquired at the lowermoft Point of the Curve, and *u* the Velocity acquired at any other Point of the Curve. Suppofe the Element of the Curve to be defcribed by this Velocity *u*, but the Element of the Bafe to be always defcribed by the conftant Velocity *a*. Then it is eafily demonftrated without any Computation, that the Element of the Ordinate being given, the Difference of the Times in which the Elements of the Curve and Bafe are thus defcribed is a *Minimum*, when the *Ratio* of thofe Elements is that of *a* to *u*; *i. e.* when the Sine of the Angle, in which the Ordinate interfects the Curve, is to the *Radius* in this *Ratio*. Suppofing therefore this Property to take place over all the Curve, the Excefs of the Time in which it is defcribed by the Body defcending alongft it, above the Time in which the Bafe is defcribed uniformly with the Velocity *a*, muft be a *Minimum*; and this latter Time being given, it follows that the Time of Defcent in this Curve is a *Minimum*. When the Gravity tends to a given Centre, fubftitute an Arc of a Circle defcribed from that Centre through the lowermoft Point of the Curve in the Place of the Bafe in the former Cafe; and the Property of the Line of fwifteft Defcent will be difcovered in the

fame

fame manner. The Nature of the Line that among all thofe of the fame Perimeter is defcribed in the leaft Time, is difcovered with great Facility, by determining from the former Cafe the Property of the Figure when the Sum or Difference of the Time in which it is defcribed by the defcending Body, and of the Time in which it would be defcribed by any given uniform Motion, is a *Minimum*; for the latter Time being the fame in all Curves of the fame Length, it follows that the Figure, which has this Property, muft be defcribed in lefs Time than any other of an equal Perimeter. The general Ifoperimetrical Problems are refolved, and the Solutions are rendered more general, with like Facility by the fame Method; which is alfo applied for determining the Property of the Solid of leaft Refiftance, and ferves for refolving the Problem, when Limitations are added concerning the Capacity of the Solid, or the Surface that bounds it.

The laft Chapter of the Firft Book treats chiefly of Gravitation towards Spheroids, of the Figure of the Planets, and of the Tides. The Author, having Occafion in thofe Inquiries for feveral new Properties of the Ellipfe, begins this Chapter by deriving its Properties from thofe of the Circle, by confidering it as the oblique Section of a Cylinder, or as the Projection of the Circle by parallel Rays upon a Plane oblique to the Circle. In this manner the Properties are briefly transferred from the one to the other, becaufe by this Projection the Centre of the Circle gives the Centre of the Ellipfe; Diameters perpendicular to each other in the Circle with their Ordinates, and the circumfcribed Square, give conjugate Diameters of the

the Ellipfe with their Ordinates, and the circumfcribed Parallelogram; parallel Lines in the Plane of this Circle are projected by Parallels in the Plane of the Ellipfe that are in the fame *Ratio*; any Area in the former is projected by an Area in the latter, which is in an invariable *Ratio* to it; and concentric Circles give fimilar concentric Ellipfes. It is likewife fhewn how Properties of a certain kind are briefly transferred from the Circle to any conic Section with the fame Facility.

After demonftrating the Properties of the Ellipfe, it is fhewn, that if the Gravity of any Particle of a Spheroid being refolved into two Forces, one perpendicular to the Axis of the Solid, the other perpendicular to the Plain of its Equator, then all Particles, equally diftant from the Axis, muft tend towards it with equal Forces; and all Particles at equal Diftances from the Plain of the Equator, gravitate equally towards this Plain; but that the Forces with which Particles at different Diftances from the Axis tend towards it, are as the Diftances; and that the fame is to be faid of the Forces with which they tend towards the Plain of the Equator.

From this it is demonftrated, that when the Particles of a fluid Spheroid of an uniform Denfity gravitate towards each other with Forces that are inverfely as the Squares of their Diftances, and at the fame time any other Powers act on the Particles, either in right Lines perpendicular to the Axis, that vary in the fame Proportion as the Diftances from the Axis, or in right Lines perpendicular to the Plain of the Equator, that vary as their Diftances from it; or when any Powers act on the Particles of the Spheroid, that may be refolved into Forces of this kind; then the
Fluid

Fluid will be every-where *in Æquilibrio*, if the whole Force that acts at the Pole be to the whole Force that acts at the Circumference of the Equator, as the Semidiameter of the Equator to the Semiaxis of the Spheroid; and that the Forces with which equal Particles at the Surface tend towards the Spheroid, will be in the same Proportion as Perpendiculars to its Surface, terminated either by the Plane of the Equator, or by the Axis. Because the centrifugal Force with which any Particle of the Spheroid endeavours to recede from its Axis, in consequence of the diurnal Rotation, is as the Distance from the Axis, it appears that if the Earth, or any other Planet, was fluid, and of an uniform Density, the Figure which it would assume would be accurately that of an oblate Spheroid generated by an Ellipsis revolving about its Second Axis.

Afterwards the Gravity towards an oblate Spheroid is accurately measured by circular Arcs, not only at the Pole, but also at the Equator, and in any intermediate Places; and the Gravity towards an oblong Spheroid is measured by Logarithms. The Gravity at any Distance in the Axis of the Spheroid, or in the Plane of the Equator produced, is likewise accurately determined by like Measures, without any new Computation or Quadrature, by shewing that when Two Spheroids have the same Centre and *Focus*, and are of an uniform Density, the Gravities towards them at the same Point in the Axis or Plane of the Equator produced, are as the Quantities of Matter in the Solids.

This Theory is applied for determining the Figure of the Earth, by comparing the Force of Gravity in

any

any given Latitude, derived from the Length of a *Pendulum* that vibrates there in a Second of Time, with the centrifugal Force at the Equator, deduced from the periodic Time of the diurnal Rotation, and the Amplitude of a Degree of the Meridian; or by comparing the Lengths of *Pendulums* that vibrate in equal Times in given unequal Latitudes; or by comparing different Degrees meafured upon the Meridian. By the beft Obfervations it would feem, that there is a greater Increafe of Gravitation, and of the Degrees of the Meridian from the Equator towards the Poles, than ought to arife from the Suppofition of an uniform Denfity. Therefore the Author fuppofes the Denfity to vary from the Surface towards the Centre; and, in feveral Cafes he has confidered, he finds that a greater Denfity towards the Centre would account for a greater Increafe of Gravitation towards the Poles, but not for a greater Increafe of the Degrees of the Meridian; and that the Hypothefis of a lefs Denfity towards the Centre would account for the latter, but not for the former, fuppofing (after Sir *Ifaac Newton)* the Columns of the Fluid to extend from the Surface to the Centre, and there to fuftain each other. On this Account he determines the Gravitation towards the Earth, when it is fuppofed to be hollow with a *Nucleus* included, according to the Hypothefis advanced by Dr. *Halley,* with the Difference of the Semidiameters that might arife from fuch a Difpofition of the internal Parts. But in this Cafe, and when the Denfity is fuppofed variable, the fpheroidical Figure is only affumed as an Hypothefis. He adds, that by imagining the Denfity to be greater in the Axis than in the Plain of the Equator at equal Di-

Diſtances from the Centre, an Hypotheſis perhaps might be found, that would account for moſt of the *Phænomena*; but that a Series of many exact Obſervations is requiſite, before we can examine with any Certainty the various Suppoſitions that may be imagined concerning the internal Conſtitution of the Earth. This Doctrine is likewiſe applied for determining the Figure of *Jupiter*.

It follows from the ſame Theorem, that if we ſuppoſe the Earth to be fluid, and abſtract from its Motion upon its Axis, and the Inclination of the right Lines in which its Particles gravitate towards the Sun or Moon, the Figure which it would aſſume in conſequence of the unequal Gravitation of its Particles towards either of thoſe Bodies would be accurately that of an oblong Spheroid having its Axis directed towards that Body. The Aſcent of the Water, deduced from this Theorem, agrees nearly with that which Sir *Iſaac Newton* found, by computing it briefly from what he had demonſtrated concerning the Figure of the Earth. Several Obſervations are ſubjoined concerning the Tides, and the Cauſes which may contribute to increaſe or diminiſh them, particularly the Inequality of the Velocities with which Bodies revolve about the Axis of the Earth in different Latitudes.

This Chapter concludes by demonſtrating briefly, that if the Attraction of the Particles decreaſed as the Cube of their Diſtance increaſes, or in any higher Proportion, then any Particle would tend towards the leaſt Portion of Matter in Contact with it, with a greater Force than towards the greateſt Body at any Diſtance, how ſmall ſoever from it. The true

Law

Law of Gravity is better adapted for holding the Parts of each Body in a proper Union, while it perpetuates the Motions in the great Syſtem about the Sun, and preſerves the Revolutions in the leſſer Syſtems nearly regular; and the Author concludes with obſerving, that a remarkable geometrical Simplicity is often found in the Concluſions that are derived from it.

An Account of Book II. *will be given in the next* Tranſaction.

VII. *De* Calculo *prægrandi a* Muliere *cum* Urina *excreto Obſervatio Dⁿⁱ* Antonii Le-protti, *R. S. S.* Pont. Max. *Archiat. per Ab-batem* Didacum de Revillas, *R. S. S. ad D.* Smart Lethieullier *R. S. S. tranſmiſſa.*

Romæ, pridie Cal. *Januarii An.* 1743.

QUÆ quindecim abhinc annis Vidua eſt Mulier pauper quinquageſimum agens annum ſumma urinæ difficultate per quadraginta Menſes laborabat; quum nocte ei ſupervenit mictus, imo ſinceri cruoris profluvium ad tres circiter libras; ſimulque Lapis, ejus, quæ in adjecta Figura deſcribitur, formæ & molis, extruſus eſt, cujus pondus jam exſiccati uncias duas & grana novem ac viginti exæquat *. Mulier autem ingenti per eos menſes gravitatis ſenſu, aſſiduoque dolore, ad Veſicæ cervicem afficiebatur ſive cubans ſive erecta; nunc autem ſolido exacto menſe, ea invita, prodere pergunt cum ſanie urinæ. *Vide* Fig. II. *in* TAB. I.

---

unc. den. gr.
* i. e. 1 : 17 : 4. TR.

VIII. *De-*